Radiative transfer with GYOTO

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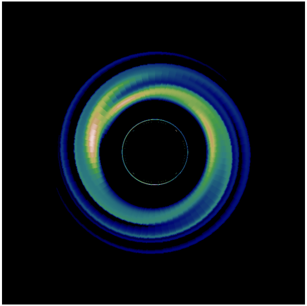
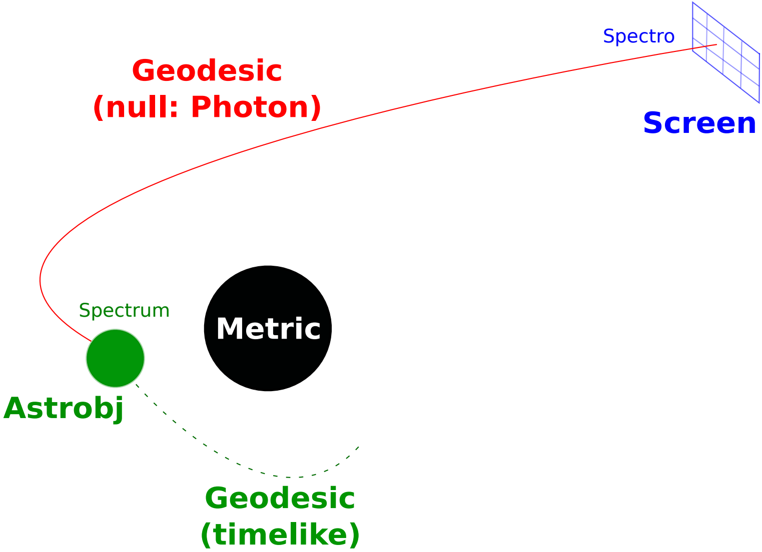
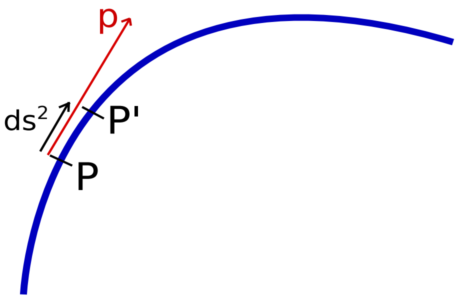


Contents

Scenery:


Spectro
Geodesic
(null: Photon)
Screen
Spectrum
Metric
Astrobj
Geodesic
(timelike)



- Trajectory: defined by tangent vector **p**, $p^\alpha = \dot{x}^\alpha$
- Trajectory of particles subject to gravity alone = **geodesic**

  $\boxed{\ddot{x}^\alpha + \Gamma^\alpha_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = 0}$; *equation of geodesics*
- $\Gamma^\alpha_{\mu\nu}$: encodes how geodesic is bent

Radiative transfer

$$\boxed{\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = j_\nu - \alpha_\nu\, I_\nu} \quad (1)$$

$$\begin{aligned} j_\nu &= \textit{emission} \\ \alpha_\nu &= \textit{absorption} \end{aligned} \quad (2)$$

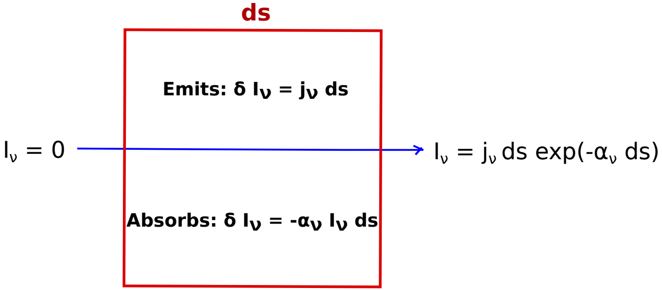
- Equation above solved in *emitter's frame*
- Then use frame invariance of I_ν/ν^3:

$$I_\nu^{\mathrm{obs}} = \left(\frac{\nu^{\mathrm{obs}}}{\nu^{\mathrm{em}}}\right)^3 I_\nu^{\mathrm{em}}$$

Radiative transfer

$$\boxed{\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = j_\nu - \alpha_\nu\, I_\nu} \tag{3}$$

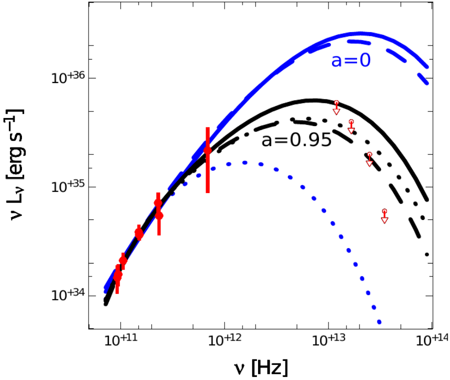
$$\begin{aligned} j_\nu &= \textit{emission} \\ \alpha_\nu &= \textit{absorption} \end{aligned} \tag{4}$$

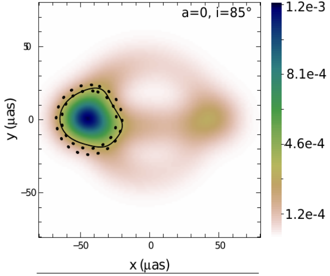


Integration methods

- Geodesic: Adaptive-step Runge-Kutta order 7/8, implemented in `boost` C++ library
- Radiative transfer: Cut geodesic inside emitting object in "small" pieces

Example: accretion torus at Sgr A* / Kerr





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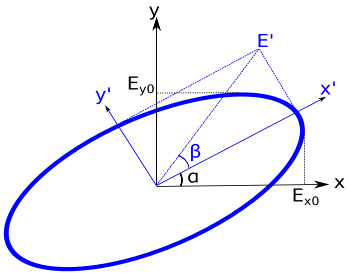

Stokes parameters in (E_x, E_y)

$$\begin{aligned} I &\equiv E_{0x}^2 + E_{0y}^2, \\ Q &\equiv I \cos 2\alpha \, \cos 2\beta \\ U &\equiv I \sin 2\alpha \, \cos 2\beta, \\ V &\equiv I \sin 2\beta \end{aligned} \tag{5}$$

Curved-spacetime specific

- Polarization frame (E_x, E_y) rotated due to geodesic bending
- So some work to do even after living the emission region...

Radiative transfer

$$\frac{\mathrm{d}I}{\mathrm{d}s} = -\alpha I + j \;\rightarrow\; \delta I(s) = j(s)\,\delta s \exp\left(-\alpha(s)\,\delta s\right)$$

Polarized radiative transfer

$$\frac{\mathrm{d}\mathbf{I}}{\mathrm{d}s} = -\mathbf{K}\mathbf{I} + \mathbf{J} \;\rightarrow\; \delta\mathbf{I}(s) = \mathbf{O}(\delta s)\mathbf{J}(s)\delta s$$

$$\mathbf{K} = \begin{pmatrix} \alpha_I & \alpha_Q & \alpha_U & \alpha_V \\ \alpha_Q & \alpha_I & r_V & -r_U \\ \alpha_U & -r_V & \alpha_I & r_Q \\ \alpha_V & r_U & -r_Q & \alpha_I \end{pmatrix}; \qquad \mathbf{J} = \begin{pmatrix} j_I \\ j_Q \\ j_U \\ j_V \end{pmatrix}. \tag{6}$$

Landi Degl'Innocenti & Landi Degl'Innocenti (Sol. Phys., 97, 239; 1985)

$$\mathbf{O}(\delta s) = \text{frightening analytical expression} \tag{7}$$

So rather similar to the unpolarized treatment, but much more cumbersome...